# APPENDIX 2 - EXPONENTIAL NOTATION AND LOGS

**Exponential notation.**
This is a convenient way of representing numbers which are very large or very small. For example, while it is convenient to write ten as 10 or one hundred as 100, larger numbers such as one million are often better written as $10^6$.
When a number is written in this form, it is called an exponential and it consists of a base (10) and an exponent (the power to which 10 is raised). Thus one million is represented as the base 10 raised to the power 6 when written as $10^6$. This is really $1 \times 10^6$, the 1 being called the pre-exponential term, but the 1 is often deleted.

exponent
↙
Thus, in exponential notation, $1 \times 10^6$
↗ ↑
pre-exponential term    base

To deduce the exponent, count the number of places that the decimal point must be moved to the left until the number becomes 1. *Note that increasing the exponent by 1 actually increases the number by a factor of 10.*

Examples: Starting with 1,000 = $10^3$,
10 x 1000 = 10,000 = $10^4$ and
10 x 10,000 = 100,000 = $10^5$ .
1,000,000,000,000 = $10^{12}$

Note that any number raised to power 0 = 1, so $10^0 = 1$.
[See the next section for the proof of this.]

For numbers where the pre-exponential term is not 1, the pre-exponential term must be shown. Thus 120,000,000 can be written as $120 \times 10^6$ or $12 \times 10^7$ or as $1.2 \times 10^8$. In scientific notation, the pre-exponential term is usually written as less than 10 so that in this example it would be $1.2 \times 10^8$ or $1.20 \times 10^8$ or $1.200 \times 10^8$ etc, depending on how many significant figures one wishes to include .

Examples: 5600 = $5.6 \times 10^3$ (2 significant figures)
246000000 = $2.46 \times 10^8$ (3 significant figures)

If more significant figures are required, they are included as zeros appended to the pre-exponential term. The exponent when it is an integer such as in the above examples is only a scale factor and gives no indication of the number of significant figures. Using the example above, $5.60 \times 10^3$ is 3 significant figures and $5.600 \times 10^3$ is 4 significant figures.

Similarly, small numbers such as one tenth are conveniently written as 0.1 for example. However a number such as one millionth (0.000,001) may be written as $10^{-6}$ or, more correctly, as $1\text{x } 10^{-6}$, with advantage. A number such as 0.000,000,12 can be written conveniently as $12 \text{ x } 10^{-8}$ or as $1.2 \text{ x } 10^{-7}$. The exponent required can be deduced by counting the number of places the decimal point must be moved to the right to obtain the pre–exponential component used and a minus sign must prefix the exponent.

Examples: 0.0034 = $34 \text{ x } 10^{-4}$ (or $3.4 \text{ x } 10^{-3}$)
0.000,000,77 = $77 \text{ x } 10^{-8}$ (or $7.7 \text{ x } 10^{-7}$)

**Illustrative Problem 1:**
There are 602,000,000,000,000,000,000,000 atoms of carbon in 12 grams of carbon. Write this number using exponential notation.

Solution: No decimal point is shown and in this situation it is understood to be at the extreme right of the number. Moving the decimal point to the left 23 places gives 6.022 as the pre-exponential term (to 4 significant figures) and $10^{23}$ as the exponential term. i.e. $6.022 \text{ x } 10^{23}$ atoms.

Note that apart from the inconvenience of the length of writing the number 602200000000000000000000, the precision of this experimentally determined quantity is certainly not to 25 significant figures! Consequently, another advantage of using exponential notation is that it allows one to trim the pre-exponential term to an appropriate number of digits corresponding to the accuracy of the number. In this case, the best experimental data gives the number as $6.022136736 \text{ x } 10^{23}$.

**Illustrative Problem 2:**
Diamonds consist of pure carbon. If 12.0 grams of carbon contains $6.022 \text{ x } 10^{23}$ carbon atoms, how many carbon atoms are in a diamond which weighs 1.20 grams?

Solution: Let the number of carbon atoms be n.

$$\text{Number of carbon atoms in 1.0 g of carbon} = \frac{6.022 \text{ x } 10^{23}}{12.0}$$

$$\therefore \text{ the number in 1.20 g } = \frac{6.022 \text{ x } 10^{23}}{12.0} \text{ x } 1.20 = 0.602 \text{ x } 10^{23} \text{ atoms}$$

$$\text{or } 6.02 \text{ x } 10^{22} \text{ atoms.}$$

**Practice Questions.**
Convert each of the following to the exponential form to base 10. Give 3 significant figures in the pre-exponential term in each case.

| | | | |
|---|---|---|---|
| 1. | 441000000000 | 2. | 3750000000000000000000 |
| 3. | 0.00000000000000127 | 4. | 0.000580 |
| 5. | 100000000000000000000000 | 6. | 0.00264 |

Answers. 1. $4.41 \times 10^{11}$ 2. $3.75 \times 10^{21}$ 3. $1.27 \times 10^{-15}$

4. $5.80 \times 10^{-4}$ 5. $1.00 \times 10^{23}$ 6. $2.64 \times 10^{-3}$

**Multiplication and Division Using Exponentials.**

**(a) Multiplication**
Example: $10^6 \times 10^3$
Rule: Provided the exponentials have the same base, **add** the exponents of the numbers to be multiplied.

Solution: $10^6 \times 10^3 = 10^{(6+3)} = 10^9$

**(b) Division**
Example: $10^6 \div 10^2$
Rule: Provided the exponentials have the same base, **subtract** the exponent of the denominator from the exponent of the numerator of the numbers being divided.

Solution: $10^6 \div 10^2 = 10^{(6-2)} = 10^4$

Be careful when the exponent on the bottom is negative:

e.g. $10^8 \div 10^{-2} = 10^{(8-(-2))} = 10^{(8+2)} = 10^{10}$

**Proof that $n^0 = 1$**:
As an example, consider $2^0$.

This can be rewritten as $2^0 = 2^{(1-1)} = \frac{2^1}{2^1}$ which cancels to = 1

The same is valid for any number, n.

**Illustrative Problem 1:**
The world's population was 6 billion ($6 \times 10^9$) people as of 2000. If each person on average consumes 1000 Calories per day, how many Calories are required to feed the entire population for 100 days?

Solution:
Total calories = number of people x calories per day x number of days
$= 6 \times 10^9 \times 1.000 \times 10^3 \times 1.00 \times 10^2$
$= 6 \times 10^{(9+3+2)}$
$= 6 \times 10^{14}$ calories

**Illustrative Problem 2:**
If the total area of rainforest remaining on the planet = $1 \times 10^6$ square kilometres and the population is $6 \times 10^9$ people, how many people are there on average per square kilometre of remaining rainforest?

Solution:
Persons per square kilometre = number of people ÷ number of square kilometres
$= 6 \times 10^9 \div 1 \times 10^6$
$= 6 \times 10^{(9-6)}$
$= 6 \times 10^3$ people/square kilometre

If the pre-exponential component of more than one of the numbers is not 1, then the pre-exponential terms are multiplied or divided as a separate calculation. This is illustrated in the next example.

**Illustrative Problem 3:**
If each person creates on average $5 \times 10^3$ kg of garbage during a lifetime, what is the total garbage created by the current population of $6 \times 10^9$ people?

Solution:
Total garbage = average per person x number of people
$= 5 \times 10^3 \times 6 \times 10^9$
$= 6 \times 5 \times 10^{(3+9)}$
$= 30 \times 10^{12}$ or $3 \times 10^{13}$ kg/person.

**Illustrative Problem 4:**
If the total GDP of the world is $\$12 \times 10^{12}$, what is the average GDP per person?

Solution:
Average GDP = total GDP ÷ number of people

$$= \frac{12 \times 10^{12}}{6 \times 10^{9}}$$

$$= \frac{12}{6} \times 10^{(12-9)}$$

$$= \$2 \times 10^{3} \text{ per person}$$

**Illustrative Problem 5:**
Convert $2 \times 10^{-4}$ metre into micrometres (μm) given 1 micrometre = $10^{-6}$ metre.

Solution:
Number of micrometres $= \frac{2 \times 10^{-4}}{10^{-6}} = 2 \times 10^{(-4-(-6))} = 2 \times 10^{+2}$ micrometres

**Practice Questions**.

1. $10^4 \times 10^5 =$
2. $10^{15} \div 10^4 =$
3. $10^7 \times 10^{-3} =$
4. $6.0 \times 10^5 \div 2 \times 10^2 =$
5. $8 \times 10^8 \div 4 \times 10^{-3} =$
6. $4 \times 10^5 \div 2 \times 10^{-6} =$
7. $3 \times 10^{-10} \div 1.5 \times 10^{-4} =$
8. $6 \times 10^3 \times 2 \times 10^{-7} \div 4 \times 10^{-2} =$

Answers.

| | | | |
|---|---|---|---|
| 1. $10^9$ | 2. $10^{11}$ | 3. $10^{10}$ | 4. $3 \times 10^3$ |
| 5. $2 \times 10^{11}$ | 6. $2 \times 10^{11}$ | 7. $2 \times 10^{-6}$ | 8. $3 \times 10^{-2}$ |

**Logarithms.**
Any number can be expressed in the exponential form without a pre-exponential term (or more precisely, with 1 for the pre-exponential term). The exponent is the logarithm (log for short) to the base ten such that 10 raised to that power equals the number.
Thus $\log_{10} n$ is the power to which 10 must be raised to equal n or $10^{\log n} = n$.

e.g. $\log_{10}10 = 1$ i.e. $10^1 = 10$

$\log_{10}100 = 2$ $10^2 = 100$

$\log_{10}2 = 0.30$ $10^{0.30} = 2$

Usually the base is not shown and then it is understood to be 10.

The log of any number can most easily be obtained from basic calculators by entering the number and pressing the LOG key. Some calculators such as the Casio fx100s require the LOG key to be pressed first, enter the number and then press the = key.

**Practice Questions**.
Obtain the logs of the following numbers. Give each to 3 decimal places.
Thence express each number in the form $10^x$. What do you notice about the answers to questions 3, 4 and 5?

1. 2.483 2. 10.870 3. 194 4. 1940 5. 19400

Answers. 1. 0.395; $10^{0.395}$ 2. 1.036; $10^{1.036}$ 3. 2.288; $10^{2.288}$

4. 3.288; $10^{3.288}$ 5. 4.288; $10^{4.288}$

The answers to Q3 - Q5 increase by 1 sequentially because the numbers they represent have increased by a factor of 10 in sequence.

More basic maths revision and theory of logs as well as further information on using calculators to deal with exponentials and logs can be found on the downloads web site at
http://www.chemlab.chem.usyd.edu.au/download.htm